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THE GENERAL RELATIONS FOR FLOW IN SOLIDS AND THEIR APPLICATION TO THE PLASTIC BEHAVIOR OF TIRE CORDS

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Numerous researches on metals, rocks, glass, rubber, and textiles are cited to show that what may be regarded as *normal* creep in amorphous and polycrystalline solids conforms to the general relation

$$\varepsilon = \varepsilon_1 + at + b \log t,$$

where ε is the total strain at time t, ε_1 is a parameter interpretable as the approximate initial strain, and a, b are other parameters. A special form of this equation, having $a = 0$, has been frequently applied. Reference is made to several other studies on the same class of solids, which have established for normal relaxation the relation

$$\sigma = \sigma_1 - \beta \log t,$$

where σ is the stress at time t, σ_1 is the stress at unit time, and β is a parameter. A theoretical foundation for both equations is provided by the reaction-rate theory of plastic flow. Observations in the present study indicate that creep extension in cotton and rayon tire cords over prolonged periods of time follows the above creep equation, and, in general, is not adequately represented by an equation omitting the term at. From the presence of this term in the equation, the existence of a component of the viscous type in tire cord growth is deduced. The pattern of creep recovery in cotton tire cord appears to be set by the behavior of the cord in creep extension.

The phenomena of creep and relaxation arising from flow in solids has been studied in a wide variety of materials ranging from soft bread doughs to the rocks in the crust of the earth. Attention has frequently been called to the technical importance of these flow phenomena in rubber products, textiles (particularly the industrial textiles, such as tire cord), high polymeric plastics, metals, and numerous other materials normally accepted as "solids." Considering the great diversity of chemical composition, molecular character, and general physical behavior, it is indeed remarkable that the laws describing the general pattern of creep and relaxation in a large number of these substances take the same two analytical forms. The only characteristic which is evidently common to all of this broad group of substances is a structure which is predominantly either amorphous or polycrystalline. This observation suggests that the types of behavior in creep and relaxation common among these substances are to be associated with randomness of structure.

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Any attempt to bring experimental data on a wide variety of otherwise unrelated materials into conformity with a general law should proceed with caution. On the other hand, the scientific ideal of systematized knowledge dictates that such possible generalizations be recognized, though they should not become the means for the entrenchment of misinformation. In the field of plastic-solid flow, a dispersive trend has prevailed, new empirical equations having been advanced with no apparent attempt having been made to correlate the data with earlier, applicable equations. The benefit of a *general*, empirical law, especially if there is a concomitant explanatory *theory*, lies in its suggestion of the elementary processes giving rise to exceptional behavior in particular cases.

THE LAW OF CREEP

The deformation resulting from creep under constant load at constant temperature in a large number of materials is in conformity with the relation

$$\varepsilon = \varepsilon_1 + at + b \log t, \tag{1a}$$

where ε is the strain at time t, ε_1 is a parameter having the dimensions of strain, and a, b are parameters depending on the load, material, and ambient experimental conditions. $\varepsilon(t)$ in Eq. (1a) is to be regarded as a function describing the creep process; in general, some other function of time describes the deformation in the initial stage. Thus, while the two processes are continuous physically, Eq. (1a) is invalid at $t = 0$, and no attempt to evaluate $\varepsilon(t)$ at $t = 0$ should be made. The parameter ε_1 may be interpreted as an *extrapolated* initial strain, for if unity on the time scale used with *Eq. (1a) is taken* as the instant at which the initial strain is completed, and the insignificant contribution of at during the very brief initial stage is neglected, it will be seen that the deformation at later times behaves *as if* its initial value were ε_1. In general, ε_1 only approximates the strain *actually* occurring during the initial stage. Equation (1a) was advanced by Weaver[1/] to represent the total creep extension of various steels at constant stress and temperature in tests lasting, in some cases, five years. The parameter corresponding to ε_1 was negative, however, and was not given the interpretation outlined above.

An approximate relation of a type similar to Eq. (1a), omits the term at, thus implicitly placing $a = 0$, and giving

$$\varepsilon = \varepsilon_1 + b \log t. \tag{1b}$$

[1/] S. H. Weaver, Trans. A.S.M.E. *58*, 745 (1936).

The creep component of the total deformation is here represented solely by a logarithmic function of the time. The possibility of so representing creep appears to have been first recognized by Boltzmann[2] in 1874. He used an equation of the type of (1b) to express the torsional deformation in a filament as a function of time.

In order to define the function $\varepsilon(t)$ so that it will be continuous at $t = 0$, the argument of the logarithmic term may be written in the binomial form $(ct + 1)$. Applying this modification, Eqs. (1a) and (1b) become

$$\varepsilon = \varepsilon_1 + at + b \log (ct + 1), \tag{1c}$$

and

$$\varepsilon = \varepsilon_1 + b \log (ct + 1), \tag{1d}$$

where c is a parameter having the dimension of inverse time. For moderately large values of ct, however, the relation

$$b \log (ct + 1) \approx b \log c + b \log t$$

holds very closely. Hence, if ε_1 in Eqs. (1a) and (1b) is assumed to include the constant term $b \log c$, no distinction between Eqs. (1a) and (1c) or (1b) and (1d) can be made on the basis of conformity with experiments in which $ct \gg 1$. For this reason, the form $\log (ct + 1)$ appears to be more comprehensive than is usually necessary for the representation of creep. Equation (1c), which may be regarded as the most general law of creep under constant load, has not been found in the literature. Equation (1d), however, has been used, as indicated below.

Since its application to creep in steel by Weaver,[1] Eq. (1a) has been employed to represent the slow yield of limestone, and of a single crystal of halite under compression,[3] as well as the creep in molded phenolic plastics.[4] Other applications of this equation have been cited by Findley.[5] Furthermore, there are in the literature several researches on creep which have yielded results evidently in conformity with Eq. (1a); the reports, however, do not cite the equation. Among these may be mentioned the tensile tests of Trouton and Rankine[6] and Andrade[7] on lead, those of Bailey[8] on molybdenum steel at elevated temperatures, and the three-hour compressive tests of Lyons[9] on copper and lead. The data published on creep under tension in eight organic plastics of various types[10-12] appear to be in accord with Eq. (1a) when moderate deforming stresses are employed. Examination of the results of Findley[5,12] indicates that under high stresses the creep in cellulose acetate proceeds as if a in Eq. (1a) had a negative value, which is a rather unacceptable condition. Griggs[3] has noted a similar behavior in creep in shale. The physical interpretation of Eq. (1a), however, provides an explanation of this behavior at high stress.

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[2] L. Boltzmann, Pogg. Ann. 7, 624 (1876), Sitz. K. Akad. Wiss. Wien, Math.-Naturwiss. Classe 70, 275 (1874).

[3] D. Griggs, J. Geol. 47, 225 (1939).

[4] D. Telfair, T. S. Carswell, and H. K. Nason, Mod. Plastics 21, 137 (Feb. 1944).

[5] W. N. Findley, *Symposium on Plastics* (Philadelphia, A.S.T.M., 1944), p. 118.

[6] F. T. Trouton and A. O. Rankine, Phil. Mag. [6] 8, 538 (1904).

[7] E. N. Andrade, Proc. Roy. Soc. A84, 1 (1910); A90, 329 (1914).

[8] R. W. Bailey, J. App. Mech. (Trans. A.S.M.E.) 58, (A)1 (1936).

[9] W. J. Lyons, J. App. Phys. 9, 641 (1938).

In the field of textiles, several studies have yielded results which are amenable to expression by means of Eq. (1a). Among these are the long duration creep results of Leaderman[13] on silk, and viscose and acetate rayon filaments. The data on single Nylon filaments, however, present an anomaly with reference to Eq. (1a) which resembles that found in cellulose acetate under high stress. The creep results of Busse and co-workers[14] on rayon tire cord suggest that both the parameters a and b increase with temperature. In the study of Dillon and Prettyman,[15] creep in cotton and rayon cords, at three different temperatures, is represented by straight lines on an elongation vs. logarithmic-time graph, indicating a value of zero for a. However, the curves found by Dillon and Prettyman for Nylon cord show a definite upturn at later times. In the experimental portion of the present study, results on tire cord creep extending into thousands of minutes were obtained. For these results Eq. (1a), as will be shown, provides a good representation.

The creep of soft copper[16] under constant tensile load, as well as that of viscose yarns,[17] has been shown to be in accord with Eq. (1d), the coefficients b and c being functions of certain material characteristics and experimental conditions. The omission of the constant rate term at from the equation, as made in Eqs. (1b) and (1d), appears to be applicable only in short duration tests, in which the viscous type of flow represented by at has little opportunity to become appreciable. Equation (1b) has been used to represent tensile creep in rubber, glass, and annealed copper, platinum, silver, and gold wires.[18] It has also been applied to creep results on cotton and rayon fibers in tension[19] and lead cylinders in compression[20] in thirty-minute tests. That creep in soft iron, also, is not a direct function of the time, but a function of the logarithm of the time, has been noted.[21]

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[10] B. Chasman, Mod. Plastics 21, 145 (Feb. 1944).

[11] J. Delmonte and W. Dewar, Mod. Plastics 19, 73 (Oct. 1941).

[12] W. N. Findley, Mod. Plastics 19, 71 (Aug. 1942).

[13] H. Leaderman, Elastic and Creep Properties of Filamentous Materials and Other High Polymers (Washington, Textile Foundation, 1943).

[14] Busse, Lessig, Loughborough, and Larrick, J. App. Phys. 13, 715 (1942).

[15] J. H. Dillon and I. B. Prettyman, J. App. Phys. 16, 159 (1945).

[16] A. Nadai and E. A. Davis, J. App. Mech. (Trans. A.S.M.E.) 58, (A)7 (1936).

[17] J. J. Press, J. App. Phys. 14, 224 (1943).

[18] P. Phillips, Proc. Phys. Soc. London 19, 491 (1905).

[19] R. L. Steinberger, Textile Research 6, 325 (1936).

[20] W. J. Lyons, J. App. Phys. 10, 651 (1939).

[21] H. Cassebaum, Ann. d. Physik 34, 106 (1911).

THE LAW OF RELAXATION

The relaxation of stress at constant deformation also has been studied in a variety of materials. The general, empirical relation, for behavior at constant temperature, may be written

$$\sigma = \sigma_1 - \beta \log \underline{t}, \qquad (2a)$$

where σ is the stress at time $\underline{t}$, σ_1 is a parameter interpretable as the initial stress, with the same limitations as were put on the interpretation of ε_1, and β is a parameter depending on the initial stress, the material, and the particular temperature of the experiment. A variant of Eq. (2a) has been announced, and we give it forthwith:

$$\sigma = \sigma_1 - \beta \log (\gamma \underline{t} + 1), \qquad (2b)$$

where γ is a parameter like $\underline{c}$.

Equation (2b), which, like Eq. (1c), is continuous at $\underline{t} = 0$, was given in 1904 by Trouton and Rankine[6] for the decay of longitudinal stress in lead wire. In the same article they indicate that Eq. (2a) fits the data on the relaxation of torsional stress in a lead wire. In subsequent research, Eq. (2b) was applied to stress relaxation under constant strain in concentrated gels of gelatine,[22] and recently it has been applied to results on rayon yarn.[17] It has been observed that Eq. (2a) represents the decay of stress in rubber under tension.[18] The graphical plots of relaxing stresses in rubber against log t yield straight lines.[23] Equation (2a) has also been found to hold for relaxation in cotton, rayon, and Nylon tire cords at various temperatures.[24]

No equation for relaxation containing a viscous-flow term such as $\underline{at}$ in Eq. (1a) has been found in the literature. There are some data, such as those of Trumpler[25] on relaxation at constant elongation in steel, which seem to call for such a term (with a negative sign), but in such cases the major portion of the relaxation is satisfactorily represented by Eq. (2a).

THEORETICAL CONSIDERATIONS

At least four equations[5, 7, 26, 27] which have been advanced to relate stress σ, strain ε, and time $\underline{t}$ during flow in various solids involve power functions of $\underline{t}$ combined with coefficients having incomprehensible dimensions. In general, the power is not integral. In many of the applications, with a suitable choice of coefficients and exponents, good fits to experimental data have been made. The utility of the equations, however, is limited to convenient analytical expression, for purposes of extrapolation or prediction. Even aside from the difficulty of interpreting fractional powers of t, these empirical equations provide no insight into the physical processes occurring during plastic deformation. Equations (1a-d) and (2a, b), on the other hand, are not only interpretable in terms of gross physical effects, but find explanation in a theory of molecular mechanism.

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[22] A. O. Rankine, Phil. Mag. [6] 11, 447 (1906).

[23] A. Tobolsky and H. Eyring, J. Chem. Phys. 11, 125 (1943).

[24] H. R. Wakeham, Southern Reg. Res. Lab., private communication.

[25] W. E. Trumpler, Jr., J. App. Phys. 12, 248 (1941).

[26] A. A. Michelson, Proc. Nat. Acad. 3, 319 (1917); 6, 122 (1920).

[27] P. G. Nutting, J. Frank. Inst. 191, 679 (1921); 235, 513 (1943).

As long ago as 1904 it was recognized by Trouton and Rankine[6] that the plastic deformation of solids involves a component of the elastic type and one of the viscous type. The former consists of flow which gradually decreases with time, and is more or less recoverable on removal of the deforming load. It has been designated by a seemingly endless number of names, among which "transient flow," used here, is one. The relation of these components (represented by dashed curves) to the total deformation ε (the solid curve) during creep, is shown in Fig. 1. Such a graphical analysis may be made without reference to a specific equation. Figure 1, however, lends itself well to the identification of the terms in Eq. (1a). The term ε_1 represents the deformation occurring in the initial stage, though as indicated above, numerically the representation is merely approximate. In the second term, a is the constant rate of flow characterizing the viscous component, which persists indefinitely. The transient flow is expressed with good fidelity by the final term b log t. As Eq. (1a) indicates, the total strain at any instant during steady creep is given by the simple arithmetic sum of the components. The strain due to creep alone is $\varepsilon_c = at + b \log t$.

The processes outlined in Fig. 1 have been interpreted in terms of another well-known creep equation,[28] namely,

$$\varepsilon = \varepsilon_1 + \lambda t + \varepsilon_\infty [1 - \exp(-t/\tau)], \tag{3}$$

where λ is the constant rate of viscous flow, ε_∞ is the final, maximum strain due to transient flow, and τ is the Maxwellian relaxation time for the system. This equation was developed for a system of models of the type considered by Maxwell in the derivation of his classical equation for relaxation. Involving the evaluation of four coefficients, instead of three as does Eq. (1a), Eq. (3) is less manageable. Furthermore, as was found in the present study on tire cord, the fit to experimental data which is finally achieved is not as good as that which Eq. (1a) readily gives. Equations of the type of (3), however, have found favor, presumably because of their theoretical foundations, which until recently Eqs. (1) and (2) have lacked. Except insofar as the transient term b log t is an approximation to $\varepsilon_\infty(1-\exp(-t/\tau))$, Eqs. (1) and (2) are not derivable from Maxwell's equation. Integration of the latter equation for constant deformation leads to

$$\sigma = \sigma_1 \exp(-t/\tau), \tag{4}$$

as the expression for the relaxation of stress. This equation, however, has been notoriously poor in representing experimental data. While it may be in accord with data on a few materials, it fails for all those cases in which the corresponding empirical Eq. (2a) holds. Trouton and Rankine[6] were among the earliest to abandon Eq. (4) in favor of Eqs. (2a) or (2b). Since the Maxwell equation is empirically unsatisfactory in the simplest applications, there appears to be little reason to accredit the underlying theory, and attempt to interpret more complicated processes in terms of it, as use of Eq. (3) implicitly does.

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[28] J. M. Burgers, First Report on Viscosity and Plasticity (Committee of Academy of Sciences at Amsterdam, Amsterdam, 1939), second edition, p. 28.

It is on the transient flow that most theoretical interest in creep and relaxation has centered. It will be noted that it is in the term for this component that Eqs. (1a) and (3) differ. The transient flow reappears in relaxation processes and is represented in Eqs. (2a) and (2b) by $\beta \log \underline{t}$, or $\beta \log (\gamma \underline{t} + 1)$. Some of the earliest analytical considerations of solid flow[7,18] associated the transient part with local rupture, rotation, or reorientation of the submicroscopic elements of the material. Transient flow, as the integration of localized movements, was distinguished from the shearing process in continuing viscous flow. The later ideas considered the transient flow to be imposed on the viscous. This view is in line with the diagram of Fig. 1.

While qualitative theory has retained this picture of the general character of transient flow, the justification for the empirical, logarithmic form has been lacking. It is possible to set down a general differential equation for the process of deformation in solids, from which the initial and transient terms in Eqs. (1) and (2) can be derived and correlated. The treatment, however, is a mathematical formalism, not unlike those accompanying application of Maxwell's equation; it contributes nothing toward establishing the logarithmic form as the unique expression for specific molecular processes. The descriptive, reaction-rate theory of Eyring[29] does provide a basis for the logarithmic form in a mechanism at the molecular level. This theory leads to the following relation between stress and time:

$$\tanh \frac{B\sigma}{2} = \tanh \frac{B\sigma_1}{2} \exp\left(- ABG\underline{t}\right), \qquad (5)$$

where σ_1 is the initial stress, $\underline{G}$ is an elastic modulus, and $\underline{A}$ and $\underline{B}$ are parameters involving temperature among other factors. Tobolsky and Eyring[23] have shown that, applied to longitudinal creep under constant load, beyond the range in which the cross section is changing rapidly, Eq. (5) leads approximately to Eq. (1b). For stress relaxation at constant elongation they derive Eq. (2a).

The constant-rate term $\underline{at}$ in Eq. (1a) does not emerge from the reduction of Eq. (5), which refers to the integration of localized slipping processes, such as might occur in crystalline solids or high polymeric network-structures. These processes consist of recrystallization arising from the assumption of new equilibrium positions by displaced atoms, and the reformation of ruptured bonds between chain molecules. In the complete Eyring theory of solid flow, however, there appears an equation for the motion of the amorphous portion of linear-polymeric substances ("segment motion").

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[29] H. Eyring, J. Chem. Phys. 4, 283 (1936).

If this equation is integrated independently for constant stress, the resultant expression yields a strain which increases at a constant time-rate. The theory in its present form, however, does not employ the outright addition of this strain to that given by Eq. (5). It does not appear feasible to dismiss the constant-rate term *at* as an insignificant experimental deviation from the logarithmic law, for in some materials, as lead in creep-extension[6] measurements for instance, the deformation resulting from viscous-like flow is the most prominent component. It may be concluded that in the creep of the numerous solids which behave in accordance with Eq. (1a), there is an actual underlying constant-rate flow corresponding to the term *at*. The constant-rate, viscous-type flow may be interpreted as a slow general distorting motion within a connected, reticular system, or between such systems, through a matrix of unassociated, amorphous substance. This slow movement could conceivably take place whether the motive force were an external load, or internal restoring stresses in the networks.

As a measure of the constant rate of viscous flow in a material or product, the parameter *a* has technical importance. The comparison of values of *a* for different materials under equal loads will indicate which will offer the most resistance to continuous yielding or growth in service.

Reference was made above to the anomalous behavior of cellulose acetate strips and Nylon filaments with reference to Eq. (1a). A decelerated creep at the longer times was observed in these materials, contrary to behavior predicted by Eq. (1a). In the light of the foregoing analysis, the observed behavior indicates extensive reorientation of the chain molecules and enlargement of crystalline areas: an excessive strain-hardening effect. Such an effect might be expected in the linear high polymers, but would not be likely to be found in polycrystalline metals having a granular structure. Presumably, under the action of high stress, or because of easy susceptibility to orientation, the strain-hardening in the acetate and Nylon becomes so pronounced as to alter seriously the viscous properties of the material and depress the value of the parameter *a*. In the case of the cellulose acetate the strain over the anomalous region was not much below that at which fracture occurred. The strain-hardening in this region may be regarded as the preliminary stage of the rupture process, and abnormal effects are to be expected. This downward curvature of the *creep vs. log t* curve at high limiting values of *t* has a counterpart in the complete graph of Eq. (5).[23]

CREEP AND RECOVERY IN TIRE CORDS

The measurement in the laboratory of creep and other visco-elastic effects in tire cords provides a useful insight into the "growth" behavior of the cords in a tire in service. Creep experiments with a constant load would appear to parallel closely the conditions in a tire which lead to growth. The cords in an inflated tire on the wheel are continually under tension. The rubber encasing the cords contributes very little toward supporting the load on the tire, and limiting the plastic extension of the stressed cords. In this respect, conditions in a tire differ from those of stress-relaxation experiments of the type in which provision is made to hold the specimen at constant length.

Since the experimental phase of the present study was intended primarily as an exploration for constant-rate flow in tire cord, relaxation experiments have not been included. Relaxation experiments should, however, provide valuable data complementary to those on creep, when the plastic behavior of a particular cord is being exhaustively examined. In tensile creep, the decay of stress is accompanied by an increase in strain tending to offset the decay. By eliminating strain as a variable, the experiments at constant length provide a means for studying the stress history of a cord directly. The observed stress-relaxation in a cord is presumably a reflection of the adjustments taking place at the micellar and molecular levels, without the complication of concomitant deformation in the fiber substance. Furthermore, the glide of fiber on fiber, which is conceivable as a component of the observed creep in cords composed of staple fibers, is substantially eliminated in stress-relaxation experiments.

Creep measurements under constant load were made at room temperature on cotton tire cords of three different types and on a rayon cord. In general, for each cord sample at each of the several loads employed, the time-sequence of observations was made on five specimens. The mean values of the lengths of the specimens at selected times were used in plotting the elongation vs. time curves. The observations on the cords, which were hung in a vertical position, were made by means of a cathetometer. The experiments were conducted in an air-conditioned laboratory in which the relative humidity was about 50 percent. The results were not intended for purposes of rigorous comparison between these cords and those tested elsewhere at different times, so extensive equipment for close humidity control was not set up. In most cases, tests on individual specimens of a particular sample were staggered in such a way that observations corresponding to the same duration of creep were not made simultaneously. Thus, while moderate upward and downward drifts in the relative humidity and temperature may have had an influence on the creep behavior of individual specimens, it is believed that their effects on the measurements cancel out completely enough in the sample averages to justify the main conclusions of the present exploratory study.

The results of these measurements are shown graphically in Figs. 2 to 5. In each figure the original load, based on the cord number (or specific linear weight) of the sample before loading, is given on the left-hand side of the figure, adjacent to the appropriate curve. In Figs. 2-4 the elongation ε is expressed in percentage of the original length of the unstressed cord.

Figure 2 shows the creep in a cotton cord of 17/4/3 construction under a series of loads in which the maximum is about 60 times the minimum. This cord had been given a treatment in which it was stretched in a wet condition, and then dried while still under tension. Since the creep tests extended over a period of only thirty minutes, no constant-rate flow is discernible. The behavior conforms to Eq. (1b). The graph reveals two features having general application to creep in textile materials: viz., the initial deformation, ε_1, is clearly not proportional to the original load, or even to the initial load σ_1, nor is the value of the coefficient b, in the transient term, proportional to these quantities. In the simplified theories of creep, the quantities corresponding to ε_1 and $\underline{b}$ are often represented as being proportional to σ_1. Typically, Fig. 2 indicates the invalidity, from the empirical standpoint, of such generalizations. Dillon and Prettyman[15] have noted a similar lack of linearity between the slope $\underline{b}$ and the load, in their creep data on tire cords.

The results given in Fig. 3, for a regular wet-twisted 29/5/3 cord, cover a much longer period of creep (4320 minutes) than do those given in Fig. 2. The creep behavior here shows a consistent deviation from Eq. (1b). The broken lines are the graphs of equations of the type of Eq. (1b) determined by the method of least squares, while the solid lines are for the best fitting equations of the type of Eq. (1a). The particular numerical equations giving ε in percent are shown. The experimental points for the loads of 0.185 and 0.549 g/grex fall in better agreement with the latter curves than with the straight lines. For the remaining two loads one curve shows as good conformity with the observations as the other. All four sets of data, however, show a tendency toward greater elongations at the later time than would be predicted by the best-fitting straight lines drawn through the first eight or nine points in accordance with Eq. (1b). The constant creep rate $\underline{a}$ tends to increase with the load.

That the constant-rate component of the observed creep in Fig. 3 arises from slippage between fibers is a reasonable tentative inference. If such slippage is the cause, the constant-rate flow should be absent from creep in continuous filament cords, for here the linear coherence does not depend on inter-filament forces. To examine this point, creep observations were made on a commercial viscose rayon cord of 1100/2 continuous-filament construction. The observations, extending over nearly 13,000 minutes are summarized in Fig. 4. It will be readily seen that, though a very moderate load (710 g, or 0.290 g/grex) was used, a constant rate component is in evidence. It becomes appreciable, however, only after about 1000 minutes of creep. Up to about 1440 minutes (1 day), the elongation is well represented simply by Eq. (1b), as indicated by the dashed straight line. The continuous growth, which in this continuous-filament cord may be confidently taken as arising from viscous flow, proceeds at a slightly lower rate than in the cotton cord in Fig. 3, under a comparable load.

While the extensive experiments of Dillon and Prettyman[15] on cotton and rayon cords fail to show the upward curvature in the graphs of elongation vs. logarithmic time, the reason for this would appear to be simply the limitation on the duration of their tests. Most of their tests covered a period of about one day. As has been noted, in the present room temperature tests the constant rate flow did not become noticeable until after about 1000 minutes had elapsed; presumably such a lower time limit would apply also to elevated-temperature tests such as Dillon and Prettyman's.

In Fig. 5 are shown recovery, as well as creep data, on two cotton cords, one the 29/5/3 regular cord to which Fig. 3 refers, and the other, a 17/4/3 dual-stretched cord, treated by the process of Philipp and Conrad.[30] On completion of the creep observations, the loads were removed, except for a clamp which weighed about 30 g, and carried the index mark. The subsequent contractions of the cords were measured periodically to yield the recovery curves in Fig. 5. To condense the graphs, only the creep ε_c, i.e., the sum of the terms for the transient and constant rate flows, has been plotted. The points represent the means of the creep in each specimen, expressed as percentage of the particular initial (1-minute) length of the specimen in the creep test.

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[30] H. J. Philipp and C. M. Conrad, J. App. Phys. 16, 32 (1945).

The data on the 29/5/3 cord appear to support Eqs. (1a) and (1b) equally well, for both the creep and recovery processes, if the parameters a and b are given negative signs in the application to recovery. Similarly, the creep data on the 17/4/3 cord favor neither equation, but those for the recovery in this cord show a definite conformity to Eq. (1a). The Boltzmann super-position principle[13] does not hold for these results since very little of the initial strain was recovered when the cords were unloaded. However, it is found, in partial agreement with this principle, that a single value for the coefficient of the transient term b log t can be used in the equations for both creep and recovery in both cords, to yield curves which satisfy the data moderately well. The necessary values for the rate parameter a are not found to be the same; in one case, that of recovery in the 29/5/3 cord, a = 0, if b is given the value 0.264, which the creep data favor. A smaller value of b which, with a = 0, would give a better fit to the recovery data, would lead to a poorer fit in the creep curve. The particular equations and the corresponding curves are shown in Fig. 5.

The appearance of a constant-rate term in the expression for recovery may be deduced from the Eyring theory. In recovery there is no constant external load producing continuous flow. However, the molecular network structure, such as exists in cellulosic fibers, provides small retractive stresses, which come into play when the external load is removed. When the stresses become very small, the rate of deformation, according to the Eyring equations, becomes sensibly constant. The tendency of recovery to approach a constant rate of flow after long intervals of time has been noted also in phenolic plastics by Telfair, Carswell, and Nason.[4]

SUMMARY

As has been shown in the foregoing discussion, there is abundant evidence for accepting Eq. (1a) as the basic, empirical law for creep under constant load. Equation (1b) is merely a special form of Eq. (1a), for the case of a = 0. Equation (1a) has an adequate foundation in the reaction-rate theory of deformation, through which also it is related to the empirically-sound Eq. (2a) for stress relaxation. As with other general laws (Newton's viscosity law, for instance), there are materials which behave in an anomalous manner with respect to Eq. (1a), and there are limits outside which the equation does not hold. It evidently does not apply at high stresses near the breaking load, for instance. Notwithstanding their limitations, Eqs. (1a) and (2a), together with their underlying general theory, provide the framework of what may be regarded the normal behavior of amorphous solids. As the basic, empirical relations for normal elasto-viscous phenomena, Eqs. (1a) and (2a) would appear to be in the same category as Hooke's law for pure elastic and Newton's law for pure viscous phenomena.

The cited work of Busse and co-workers, Dillon and Prettyman, and Wakeham, as well as that reported in the present exploratory study, all point to the applicability of Eqs. (1a) and (2a) to the normal plastic behavior of tire cords. The equations have the usual limitations of empirical, semi-theoretical relations, in that they cannot be used validly to compute strains or stresses for periods 100 or 1000 times as long as that over which the constants were evaluated. When Eq. (1a), with the constants for tire cord, is used in such extrapolations, absurdly large elongations are obtained. It must therefore be concluded that the constant-rate flow, represented by the parameter a, does not go on indefinitely, but that after creep times on the order of 10^5 minutes the structure of the cord or its component fibers (as in the case of the acetate and Nylon samples mentioned previously) becomes sufficiently stabilized to counter-balance the residual forces producing the constant-rate flow.

In view of their wide verification, extending to non-textile materials, it would appear feasible to use Eqs. (1a) and (2a) as the basis for more or less standardized tests of creep and relaxation in cords and yarns. Just as a sample of steel is characterized by its moduli of tensile elasticity and rigidity at a certain temperature, so the plastic, or growth, properties of a cord may be identified by the values of the parameters ε_1, a, b, σ_1, and β under specified conditions of load, temperature, and humidity. Dillon and Prettyman[15] have taken a step in this direction with their use of the quantities "initial compliance" and "weighted creep." There can be no doubt that the utility, for purposes of comparisons, of creep and relaxation data on tire cord and other textiles, will be broadened if they are interpreted in terms of generalizations which are valid and at the same time are generally recognized. Equations (1a) and (2a) may be adopted as such generalizations.

ACKNOWLEDGMENT

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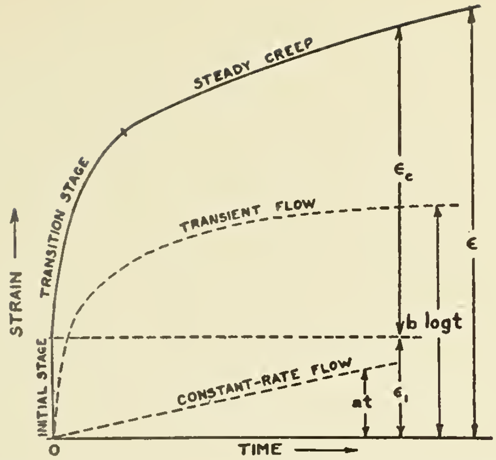

Fig. 1. Hypothetical diagram of the components of total creep deformation, as applied to terms in Eq. (1a).

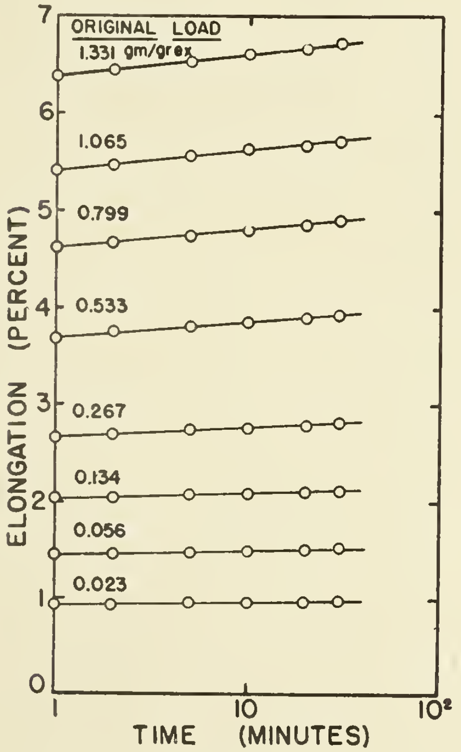

Fig. 2. Creep in a "hot-wet-stretched" 17/4/3 cotton cord.

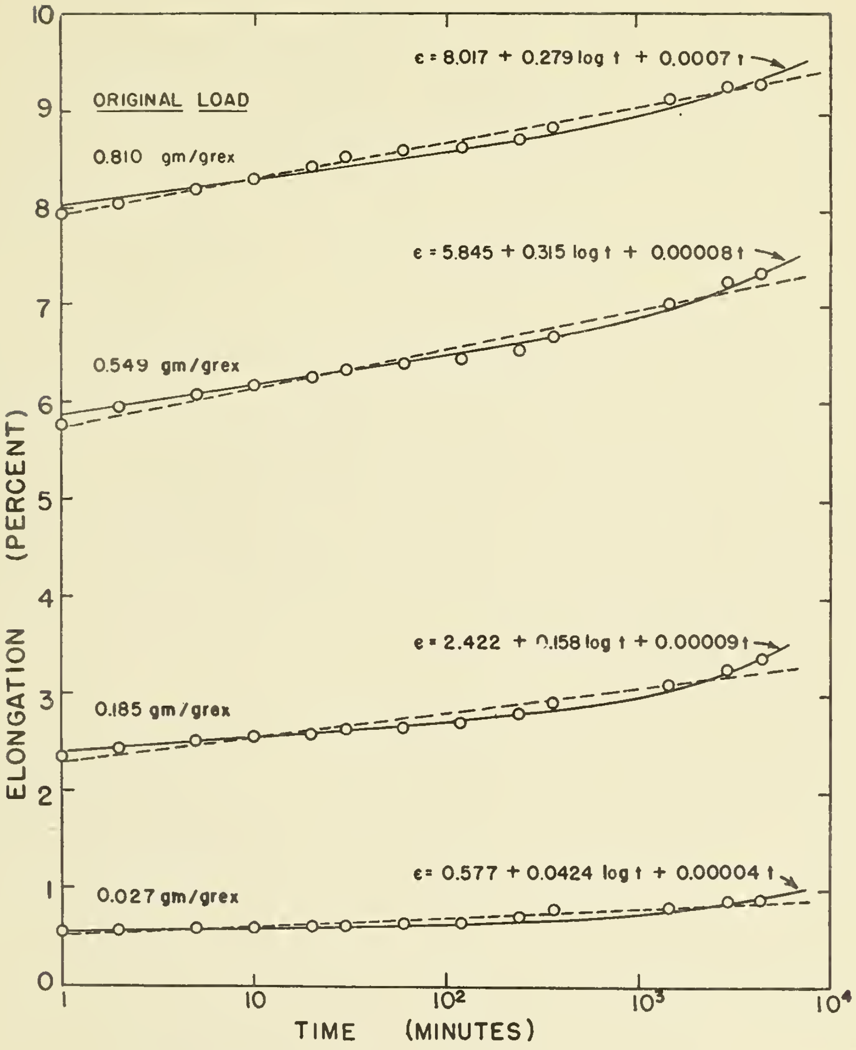

Fig. 3. Creep in a regular, wet-twisted 29/5/3 cotton cord.

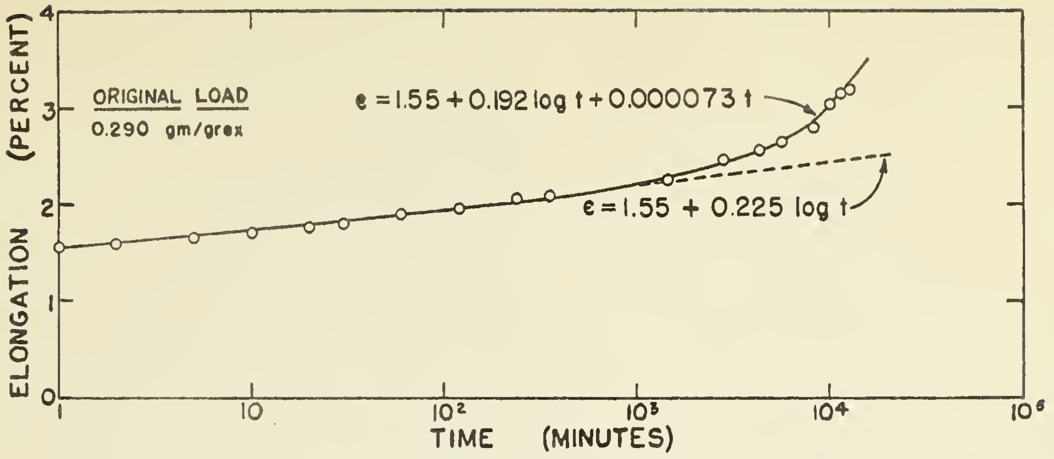

FIG. 4. Creep in a continuous-filament 1100/2 viscose rayon cord.

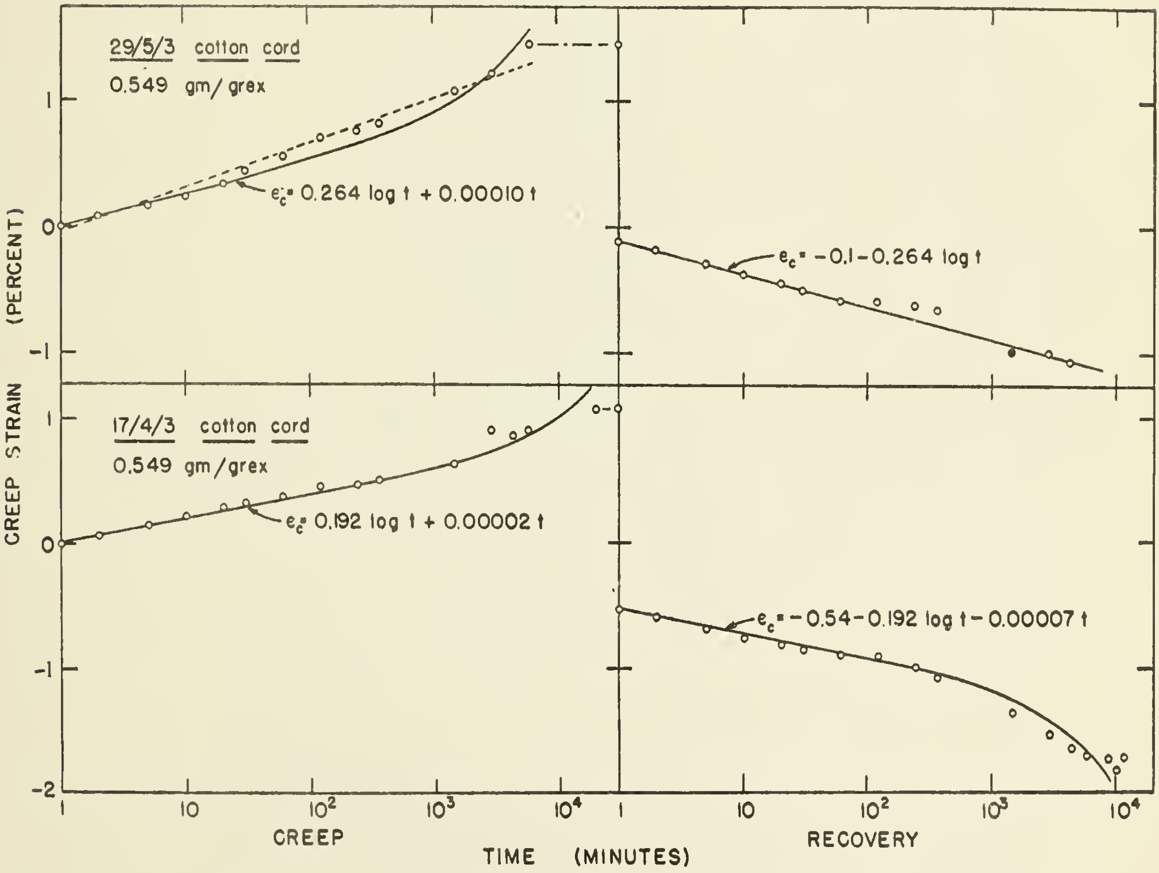

FIG. 5. Creep and recovery in a regular 29/5/3, and a dual stretched 17/4/3 cotton cord. Creep in both cords took place under an original load of 0.549 g/grex.